Numerical Simulation

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Exercise Sheet 6

Closing date **June 7, 2016**.

Exercise 1. Let $\Omega = (0,1)$ and $U = L^2(\Omega)$. Consider

$$f(u) := -\int_0^1 \cos(u(x))\,dx$$

Prove the following:

a) f is Gâteaux-differentiable.

b) f is not twice Fréchet-differentiable in $\bar{u} = 0$ with respect to the L^2-norm.

c) f is twice Fréchet-differentiable in $\bar{u}$ with respect to the L^∞-norm.

(10 points)

Exercise 2. Consider for a bounded Lipschitz domain $\Omega \subset \mathbb{R}^2$, functions $f, y_\Omega \in L^2(\Omega), u_\Gamma \in L^2(\Gamma)$, as well as $0 \leq \alpha \in L^\infty(\Gamma)$ with $\|\alpha\|_{L^\infty(\Gamma)} > 0$, and nonnegative real numbers $\lambda_\Omega, \lambda_\Gamma > 0$ the following optimal control problem:

$$\begin{aligned} \min_{(u,y)\in\in(U_{\mathrm{ad}}\times\in H^1(\Omega))} \quad J(y,u) &= \tfrac{\lambda_\Omega}{2}\|y - y_\Omega\|^2_{L^2\Omega} + \tfrac{\lambda_\Gamma}{2}\|u - u_\Gamma\|^2_{L^2(\Gamma)} \\ -\Delta y &= f \quad \text{in } \Omega \\ \partial_\nu y + \alpha y &= u \quad \text{on } \Gamma, \end{aligned}$$

where $\partial_\nu y$ denotes the outward unit normal of y, and as usual U_{ad} is given by

$$U_{\mathrm{ad}} := \{u \in L^2(\Omega) \colon u_a(x) \leq u(x) \leq u_b(x) \quad \text{a.e. in } \Omega\}$$

with $u_a, u_b \in L^\infty(\Omega)$ with $u_a(x) \leq u_b(x)$ a.e.

Construct a test example with known solution, i.e. state precise data for Ω and all appearing functions, parameters, and bounds, as well as the optimal solution triple $(\bar{u}, \bar{y}, p)$. Verify that the first order optimality conditions are fulfilled.

(15 points)