Numerical Simulation

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Exercise Sheet 10

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Exercise 1. Let X be a real Banach space, and $F\colon X \to X$ be Fréchet-differentiable with invertible derivative $F'(x)$ for all $x \in X$. Moreover, let $A, B\colon X \to X$ linear and invertible. We are looking for a root of F in X. Let $\{x^k\} \subset X$ a sequence constructed from Newton's method applied to $F(x) = 0$ with initial value $x^0 \in X$. Consider the linear transformation $y = Bx$ and the function

$$G\colon X \to X,\ G(y) = AF(B^{-1}y).$$

Prove that Newton's method is affine-invariant, i.e. a sequence $\{y^k\} \subset X$ constructed from applying Newton's method to $G(y) = 0$ with initial value $y^0 = Bx^0$, one has $y^k = Bx^k$ for all $k \in \mathbb{N}$.

(10 points)

Exercise 2. Let $\Omega \subset \mathbb{R}^n$, $n \in \mathbb{N}$, be a bounded Lipschitz domain and denote by $I = (0, T)$ a time-intervall with final time $T > 0$, and denote by $Q := \Omega \times I$ the space-time-cylinder with boudnary $\Sigma = \partial\Omega \times I$. Consider the following optimal control problem governed by a linear parabolic partial differentiable equation:

$$\min_{(u,y)\in U\times Y} J(u, y) = \frac{1}{2} \iint\limits_Q (y - y_\Omega)^2 dxdt + \frac{\lambda}{2} \iint\limits_Q u^2 dxdt$$

s.t.

$$\begin{aligned} \partial_t y - \Delta y &= u \quad \text{in } Q \\ y &= 0 \quad \text{on } \Sigma \\ y(x, 0) &= y_0 \quad \text{in } \Omega \\ a \le u(x, t) &\le b \text{ for a.a. } (x, t) \in Q, \end{aligned}$$

with $U = L^\infty(Q)$, $Y = W^{2,1}(Q)$, $y_\Omega \in L^2(Q)$, $\lambda > 0$, $a, b \in \mathbb{R}_0^+$, $a \le b$. Use the formal Lagrange-technique to derive the (expected) first-order-necessary optimality conditions in KKT-form.

(10 points)