Scientific Computing 2

Summer term 2017
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Sheet 1

Submission on **Thursday, 27.4.2017**.

Exercise 1. (multivariate quadratic polynomial)

Consider the function $f\colon \mathbb{R}^n \to \mathbb{R}, \quad f(x) = \frac{1}{2}x^T Hx + b^T x + c$.

a) Calculate the gradient ∇f and Hessian $\nabla^2 f$ for an arbitrary $H \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $c \in \mathbb{R}$ and for symmetric H.

b) Show that H can be replaced by a symmetric matrix without changing the objective function f.

c) Show that if H is positive definite, then f is strictly convex and $\lim\limits_{\|x\|\to\infty} f(x) = \infty$.

d) Rewrite the function

$$g(x_1, x_2) = 5x_1^2 + 5x_2^2 + 8x_1x_2 - 4x_1 - 2x_2 + 3$$

as $g(x) = \frac{1}{2}x^T Hx + b^T x + c$ with symmetric $H \in \mathbb{R}^{n\times n}$. Is H positive definite? If yes, what does this imply for existence and uniqueness of minima? Calculate the global minimum of g.

(6 points)

Exercise 2. (saddle point property)

Consider the minimization problem from the lecture:

$$\min f(x_1, x_2) = x_1^2 + x_2^2$$
$$x_2 \le x_1 - 1$$

Derive the corresponding Lagrangian $L(x_1, x_2, \lambda)$. Show that the solution $(\bar{x_1}, \bar{x_2}, \bar{\lambda})$ satisfies the saddle point property

$$L(\bar{x_1}, \bar{x_2}, \lambda) \le L(\bar{x_1}, \bar{x_2}, \bar{\lambda}) \le L(x_1, x_2, \bar{\lambda}) \quad \forall \lambda \ge 0, \forall x_1, x_2.$$

(4 points)

Exercise 3. (3-norm)

Calculate all minima of $f(x_1, x_2) = x_1^3 + x_2^3$ on the unit circle, using the Lagrange formalism.

(4 points)

Exercise 4. (parabolic graph)

Find the point lying on the parabolic graph $x_1^2 - 4x_2 = 0$ which minimizes the euclidean distance to $(0, 1)$. Try to solve this problem using Gaussian elimination.

- Eliminate x_1^2. What happens, and why?
- Eliminate x_2 instead.

Afterwards, use the Lagrange formalism to solve this minimization problem.

(6 points)