## Sheet 2

Submission on **Thursday, 4.5.2017**.

**Exercise 1.** (variational inequality)

Let $X \subset \mathbb{R}^n$ be convex and $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be a convex function which is continuously differentiable. Given the optimization problem

$$\min_{x \in X} f(x)\,,$$

show that $x^* \in X$ is a solution iff the variational inequality

$$\nabla f(x^*)^\top (x - x^*) \geq 0$$

holds for all $x \in X$.

(6 points)

**Exercise 2.** (ACQ and GCQ)

Determine the tangential cone $T(X, x)$ and linearized tangential cone $T_l(g, x)$ for the feasible set $X = \{g(x) \leq 0\}$ and a given $x^* \in X$. Visualize your results. Furthermore, check if the Abadie-Constraint-Qualification and Guignard-Constraint-Qualification are satisfied.

a) $g(x) = (x_2 - x_1^5,\, -x_2)^\top$, $x^* = (0, 0)^\top$

b) $g(x) = (x_2^2 - x_1 + 1,\, 1 - x_1 - x_2)^\top$, $x^* = (1, 0)^\top$

(4 points)

**Exercise 3.** (Slater CQ)

Let $f, g_1, \dots, g_m \in C^1(\mathbb{R}^n)$ be convex and $X = \{x \in \mathbb{R}^n \mid \forall i : \ g_i(x) \leq 0\}$ be the feasible set. For an optimization problem

$$\min_{x \in X} f(x)\,,$$

$X$ satisfies the Slater condition if there exists $y \in \mathbb{R}^n$ such that $g_i(y) < 0$ for $i = 1, \dots, m$. Show that the Slater condition is a constraint qualification, i.e., the Guignard-Constraint-Qualification is satisfied for all $x \in X$.

(6 points)

**Exercise 4.** (Cones)

Let $K$ be a cone and $K^\circ$ its polar cone. Prove the following statements.

a) $K$ is convex iff $K + K \subset K$.

b) $K^\circ$ is always convex and closed.

c) If $K$ is convex and closed, it follows that $(K^\circ)^\circ = K$.

(4 points)