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Sheet 4

Submission on **Thursday, 18.5.2017**.

Exercise 1. (projection)

Let $C \in \mathbb{R}^n$ be a closed, convex, nonempty set and $y \in \mathbb{R}^n$. Let further $H \in \mathbb{R}^{n\times n}$ be positive definite and consider the optimization problem

$$\min_{x\in C} f(x) = \frac{1}{2}(x-y)^\top H(x-y)\,.$$

Use an appropriate substitution to describe the solution of this problem in terms of a projection operator. Which form does the corresponding variational inequality take?

(4 points)

Exercise 2. (Minkowski inequality)

Let $1 \le p \le q < \infty$. Show the following inequality by considering an appropriate optimization problem:

$$\left(\frac{1}{n}\sum_{i=1}^n |x_i|^p\right)^{1/p} \le \left(\frac{1}{n}\sum_{i=1}^n |x_i|^q\right)^{1/q}$$

(6 points)

Exercise 3. (optimality conditions 1)

Consider the optimization problem

$$\min_{x\in\mathbb{R}^3} f(x) = x_1 + x_2^2 + x_3^3$$

with constraint

$$g(x) = 1 - (x_1^2 + x_2^2 + x_3^2) \le 0\,.$$

a) Show that all points in the feasible set satisfy a CQ.

b) Verify that $x^* = (1,0,0)^\top$ with $\lambda^* = 0.5$ satisfy the KKT conditions.

c) Compute $\nabla^2_{xx} L(x^*,\lambda^*)$.

d) Use the second order necessary optimality condition to show that x^* is not a local solution.

(4 points)

Exercise 4. (optimality conditions 2)

a) Solve the following optimization problems and check necessary and sufficient optimality conditions.

$$(i)\ \begin{cases}\min f(x_1,x_2) = (x_1-3) + x_2^2\\ x_1^2 - x_2 \le 0\end{cases} \qquad (ii)\ \begin{cases}\min f(x_1,x_2) = (x_1-2) + (x_2-1)^2\\ x_1^2 - x_2 \le 0\\ x_1 + x_2 - 2 \le 0\end{cases}$$

b) Consider the optimization problem

$$\min_{x \in \mathbb{R}^2} f(x) = x_2 + \frac{1}{2}(x_1^2 + x_2^2)$$

with constraint

$$g(x) = -x_1^2 - x^2 \leq 0\,.$$

Show that $x^* = (0,0)^\top$ satisfies the KKT-conditions and verify that $\nabla^2 f(x^*)$ is positive definite on $\mathbb{R}^2$. Is x^* a local solution? Justify your answer.

(6 points)