Scientific Computing 2

Summer term 2017
Prof. Dr. Ira Neitzel
Christopher Kacwin

universität**bonn**

Sheet 9

Submission on **Thursday, 6.7.2017**.

Exercise 1. (weak differentiability)

We consider the function $f\colon [-1,1] \to \mathbb{R}$, $f(x) = |x|$.

a) Show that f is weakly differentiable and compute its weak derivative.

b) Show that f is not twice weakly differentiable.

(4 points)

Exercise 2. (weak formulation for bilaplace)

Let $\Omega \subset \mathbb{R}^n$ be a regular domain. For $f \in L^2(\Omega)$ consider the partial differential equation

$$\Delta(\Delta u) = f$$

with boundary conditions

$$\begin{aligned} u &= 0 \text{ on } \partial\Omega \\ \partial_\nu u &= 0 \text{ on } \partial\Omega\,, \end{aligned}$$

where ∂_ν is the directional derivative with respect to the normal vector ν on $\partial\Omega$. Derive the weak formulation in $H_0^2(\Omega) = \{w \in H^2(\Omega) \mid w = 0 \text{ on } \partial\Omega,\ \partial_\nu w = 0 \text{ on } \partial\Omega\}$ for this PDE. (Hint: Use the Gauss divergence theorem / Green's identities to do the integration by parts)

(6 points)

Exercise 3. (higher regularity in 1D)

Let $I = [a,b] \subset \mathbb{R}$ and $f \in L^2(I)$. Let $u \in H_0^1(I)$ be the weak solution to the Poisson equation

$$-u'' = f$$

with Dirichlet boundary conditions. Show that u belongs to $H^2(\Omega)$.

(4 points)

Exercise 4. (finite 2D element)

Let $T \subset \mathbb{R}^2$ be the closed triangle with corners $a_1 = (0,0)^\top$, $a_2 = (1,0)^\top$, $a_3 = (0,1)^\top$. Furthermore, let $\{\phi_1, \phi_2, \phi_3\}$ be the nodal basis to this triangle, i.e., for $i = 1,2,3$ one has that $\phi_i\colon T \to \mathbb{R}$ is a linear function which satisfies $\phi_i(a_j) = \delta_{ij}$ for $j = 1,2,3$. Compute the local stiffness matrix $K \in \mathbb{R}^{3\times 3}$ with entries

$$K_{ij} = \int_T (\nabla\phi_i)^\top \nabla\phi_j\,.$$

(6 points)